Time, Logical Time and Causality

Carlos Baquero
Distributed Systems Group
Universidade do Minho

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Plan

We will try to cover a few of the many aspects of time and logical sequences of events in distributed systems:

- Time Synchronization
- Order Relations
- Logical Time and Causality
- Process Causality vs Data Causality

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- Process Causality vs Data Causality

Global Snapshots and Termination will only be covered in the next talk, so we will carefully avoid them.

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- 10^{-6} amounts to about 1 second each 12 days. Not very good.
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- Coordinated Universal Time (UTC) is a high-precision atomic time standard. It closely tracks Universal Time (UT), that maps earth rotation, by adding leap seconds when needed.

Time Synchronization

Synchronization

External Synchronization

Measures expected precision with reference to an authoritative time source.

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A system with D external synchronization also depicts $2D$ internal synchronization.

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Asynchronous

In an asynchronous system t_d now varies in range $t_m \leq t_d \leq \infty$. Apparently, the envelope is now $D = \frac{\infty - t_m}{2} = \infty$. Not a very usefull bound, but its easy to do better.

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Asynchronous System: Cristian's algorithm

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- t_r can be made smaller if we adjust for a lower bound b on message transmition time. $t_r = t_f - (t_i + b)$.
- The algorithm can be repeated until we eventually observe a t_r that gives us a "tight enough" synchronization.

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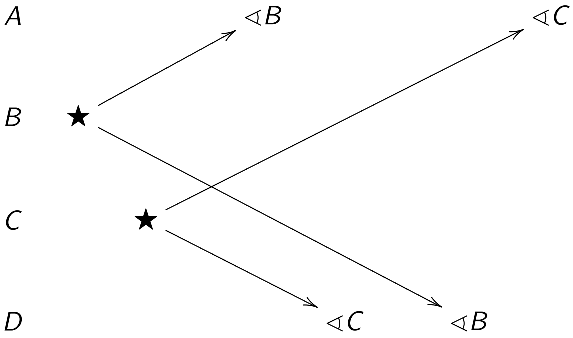
  The resulting total order is not realistic and not always usefull, since it orders events that are in fact unrelated.
- Even on physical systems real time total ordering is not always consistent for diferent observers.

Time Synchronization

Ordering Explosions: Two independent ones



Observers

While A sees $\langle B, C \rangle$, D sees $\langle C, B \rangle$.

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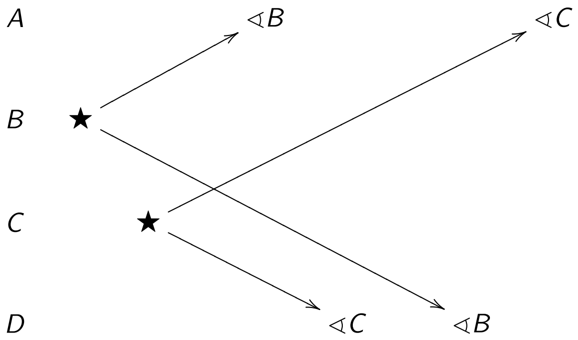

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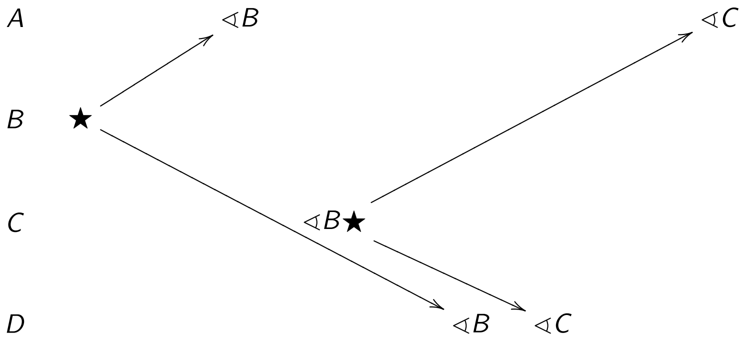
If we really need a total order (e.g. to make a replicated state machine) maybe we can give an arbitrary order to these events. As long as no one can contradict these decisions.

Time Synchronization

Ordering Explosions: One triggers the next



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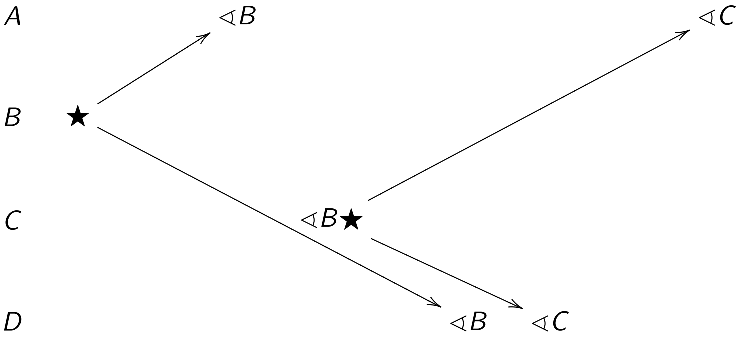

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If message propagation speed is uniform, independent observers make consistent observations of events that might be causaly related. Otherwise the world would be much more confusing ...

Order Relations

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Orders can be strict $<$ or non-strict $\leq$.

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- We have an **antichain** if $x \leq y$ iff $x = y$.

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Binary sequences

Exhibit a **prefix** ordering. Let $\mathbf{2}^*$ be the set of all finite binary strings, including $\langle\rangle$. For $x, y \in \mathbf{2}^*$ we have $x \leq y$ iff x is a finite initial substring of v. E.g. $0100 < 010011$, $010 \parallel 100$.

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Coordinatewise (pointwise) order

Let $P_1, \ldots, P_n$ be ordered sets. The cartesian product $P_1 \times \cdots \times P_n$ can define a ordered set by pointwise order:

$$(x_1, \ldots, x_n) \leq (y_1, \ldots, y_n) \Leftrightarrow (\forall i) x_i \leq y_i \text{ in } P_i.$$

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Lexicographic order

Let A, B be two ordered sets. The product $A \times B$ can have a **lexicographic order** defined by
$(x_1, x_2) \leq (y_1, y_2)$ if $x_1 < y_1$ or ($x_1 = y_1$ and $x_2 \leq y_2$).
By iteration a lexicographic order can be defined on any finite product.

Order Relations

Relations among orders

Order isomorphism

Given two partially ordered sets $\langle S, \leq_S \rangle$ and $\langle T, \leq_T \rangle$ an **order isomorphism** is a surjective (onto) total function $h : S \rightarrow T$ such that for all $u, v \in S$:
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For instance, we will see that real time total ordering is consistent with causality.

Logical Time and Causality

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h_i^k denotes an initial prefix of local history h_i containing the first k events.
The *global history* of the computation is the set $H = h_1 \cup \ldots \cup h_n$.

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Causality

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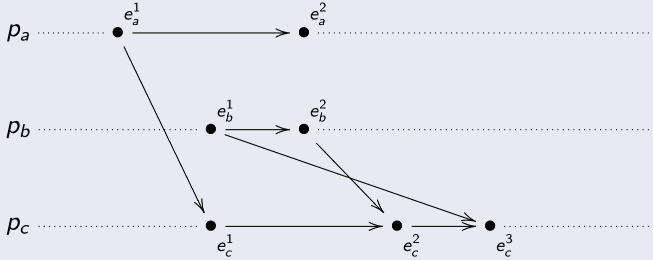
If $a \rightarrow b$ then a may have influenced b. In general we have potential causality.
On non trivial runs $\langle H, \rightarrow \rangle$ forms a partial order, and some events will be parallel $a \parallel b$ when neither $a \rightarrow b$ nor $b \rightarrow a$.

Logical Time and Causality

Preserving causal order

If we had a global time clock function $\mathcal{T} : H \rightarrow \mathbb{R}$ that would assign a real to each event. We would observe that the total order $\langle \mathcal{T}(H), < \rangle$ is consistent with $\langle H, \rightarrow \rangle$. Real time preserves the causal order.

Run



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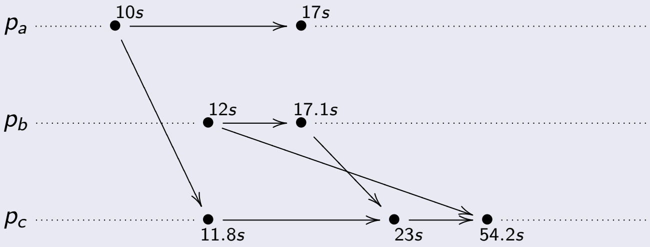

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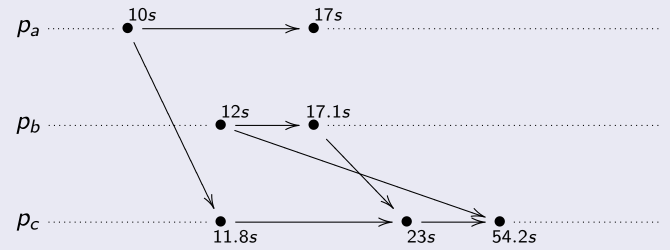

Notice that while $11.8s < 12s$ the corresponding events are parallel $e_c^1 \parallel e_b^1$ in the causal order.

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Clock condition

Being consistent with causality if often captured by a *clock condition*.

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A clock function $\mathcal{C} : H \rightarrow T$ and a ordered set $\langle T, < \rangle$ satisfies clock condition if:
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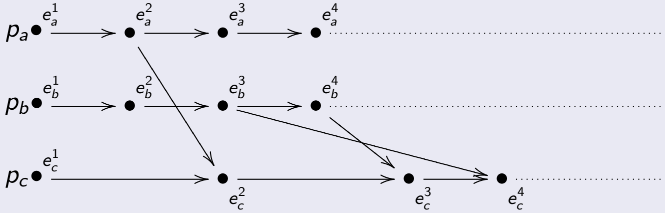
Appart from real time there are other timestamping functions that satisfy this clock condition.

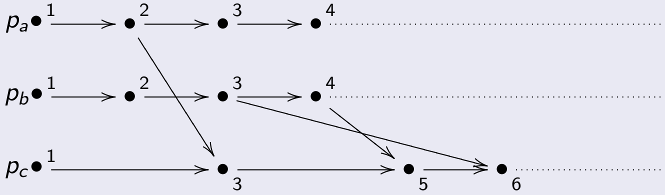
Logical Time and Causality

Lamport Time

Run



Run with timestamping consistent with causality



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Notice that while Lamport Time $\mathcal{L}$ and Real Time $\mathcal{L}$ are both consistent with causality $\langle H, \rightarrow \rangle$, the mutual relation between $\mathcal{L}$ and $\mathcal{T}$ is not tipically consistent in non trivial runs.

Logical Time and Causality

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We can further refine Lamport Time in order to obtain an injective function $\mathcal{L}^t$ that assigns a consistent total order to all events in H. It suffices to consider the lexicographic order on the pair formed by the Lamport Time and the process number.

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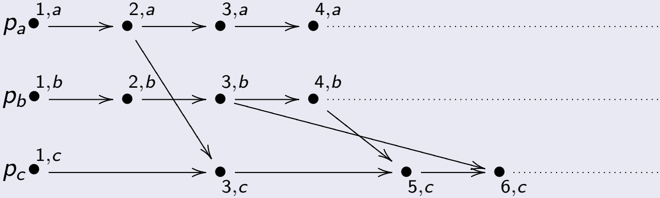
Here, since processes have letters we assume the alphabetic order.



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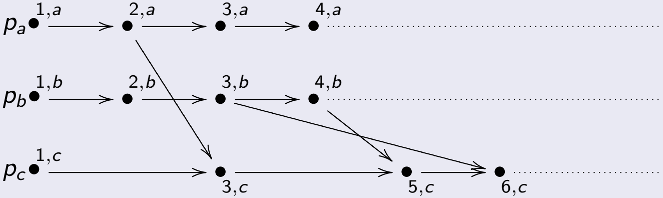

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This total order is usefull in many distributed algorithms (e.g. Lamport mutual exclusion algorithm), but it orders more events than causality. For other algorithms we need to capture causality precisely.

Logical Time and Causality
Characterizing causality

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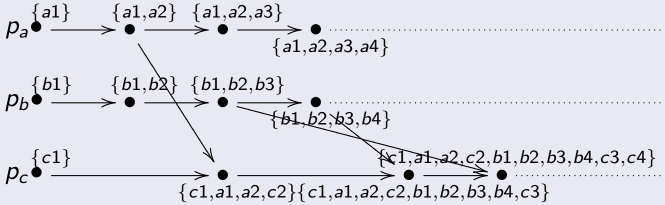

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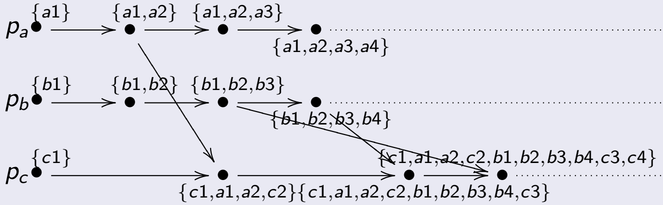

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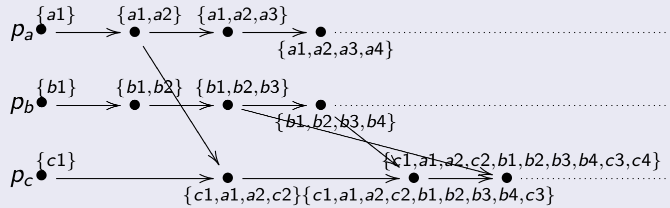

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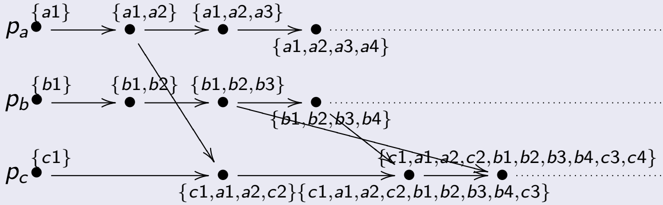

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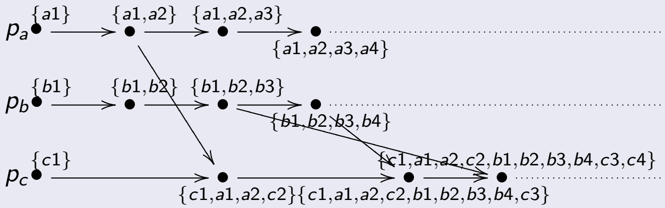

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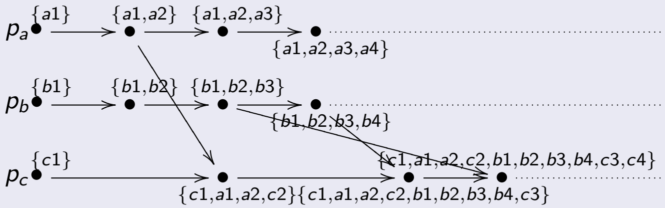

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This can be solved by noticing that for all k and i and a causal history $\mathcal{C}_x$: if $e_i^k \in \mathcal{C}_x$ then $\{e_i^1, \ldots, e_i^{k-1}\} \subseteq \mathcal{C}_x$.

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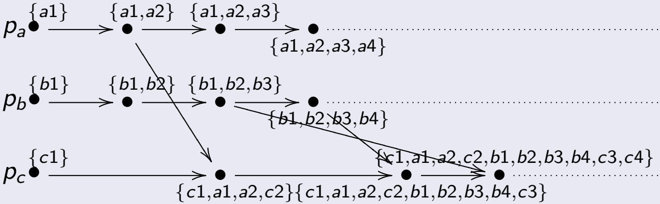

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Consequently one only needs to register the index of the last event from each process.

Logical Time and Causality

Vector Clocks

Vector clocks are compressed causal histories. $\mathcal{V} : H \longrightarrow \mathbb{N}^n$ where n is the number of processes. They can be represented in a vector or as mappings from process names to integers.

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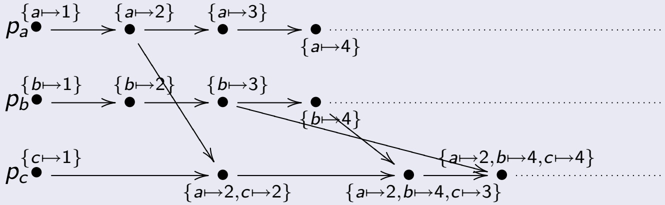

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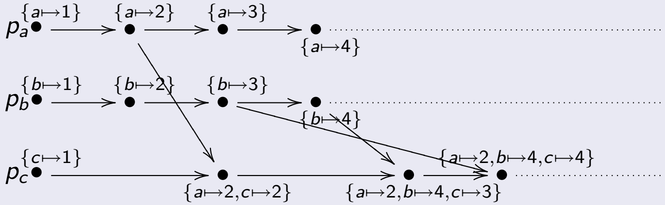
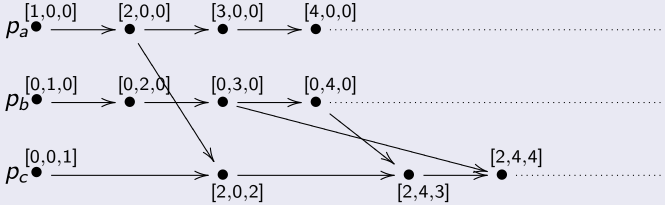


Vector clocks are used in many distributed algorithms. E.g. causal delivery of messages, an extension of FIFO delivery. They can be used as long as processes have unique ids. A total order on ids is only a convenience (trivially obtained from unique ids).

Logical Time and Causality
Vector Clocks

Run tagged with vector clocks



The cordinatewise (pointwise) order on version vectors characterizes causality. They define identical partial orders.
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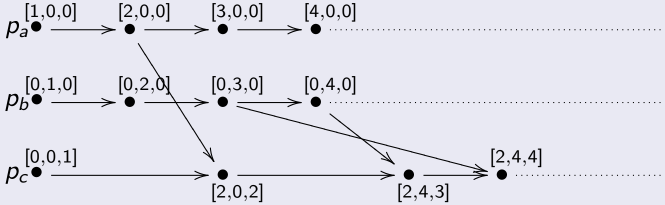

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$[2, 0, 0] < [2, 0, 2]$ but $[0, 4, 0] \parallel [2, 0, 2]$
Complexity is $O(N \log E)$ and $\mathcal{V}$ is known to be the most concise timestamping mechanism for process causality tracking.

Process Causality vs Data Causality

- Causality is formed as relevant events are colected in a run.

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- The relevant events are *update* events on the replica state.
- If each update event is distinguished, two replicas that know the same set of update events are equivalente.

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Data causality is at the core of version control systems, replicated file systems, partitioned operation and optimistic replication in general.

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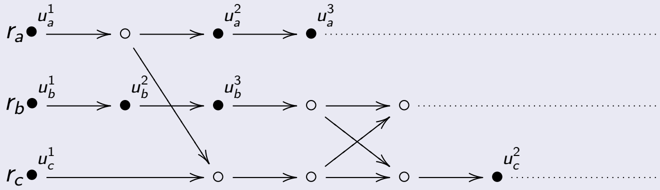

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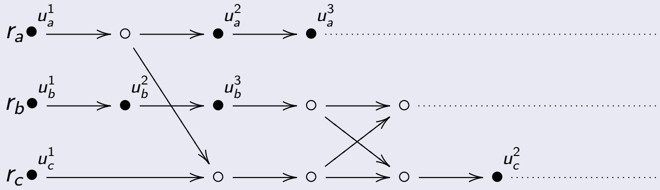
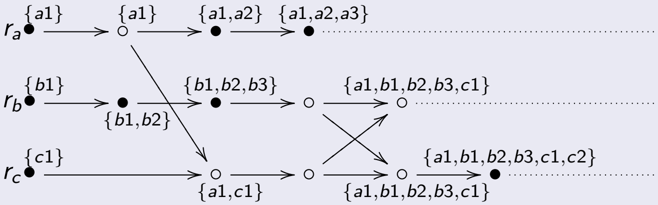


This run includes sending and receiving of messages but causality tracking can ignore these events.

Data Causality

Causal histories

Run with causal histories



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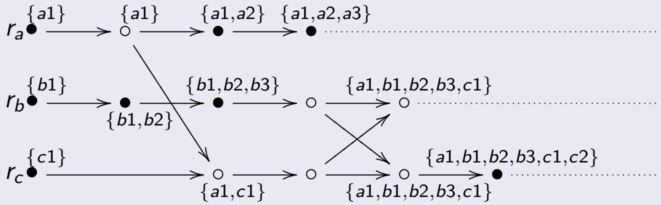

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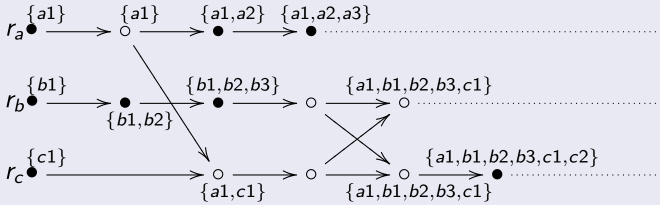

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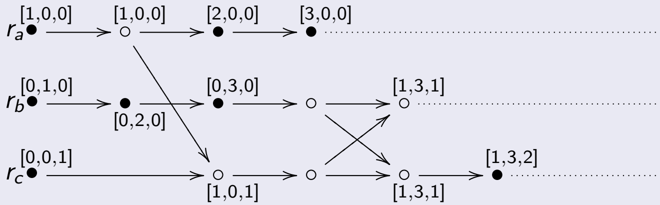
At the end of this run we have the following relations among replicas, as observed by set inclusion:

$r_a \parallel r_b$ and $r_a \parallel r_c$ and $r_b < r_c$.

Data Causality

Version vectors

Run with version vectors



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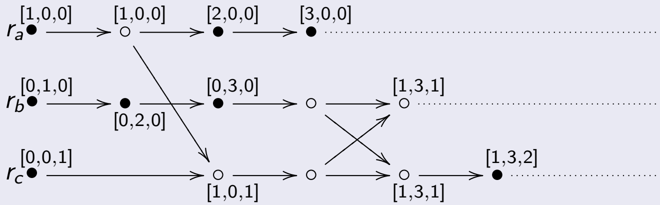

Both version vectors and causal histories characterize data causality.
Are version vectors the most concise representation of data causality?

Data Causality

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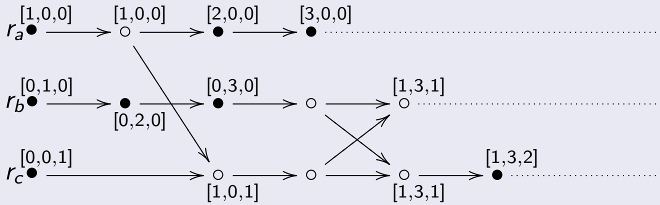

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Data Causality
Version vectors

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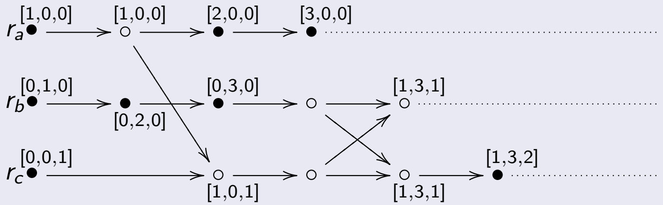

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In fact, no, altough it looks like.
Altough we have a unbounded number of update events in data causality one is only concerned about the order among existing replicas. Those forming the frontier of the run.

Data Causality

Frontier configurations

With two replicas the following cases are possible:

- $r_a = r_b$
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From "On the computer enumeration of finite topologies" they are found to be
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Data Causality

Frontier configurations

With two replicas the following cases are possible:

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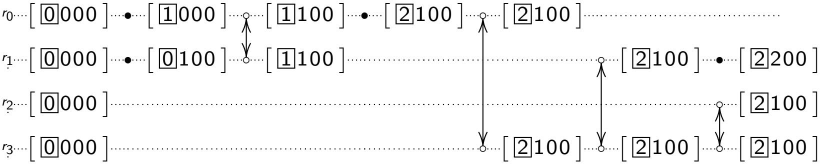
This is possible with bounded version vectors.

Data Causality

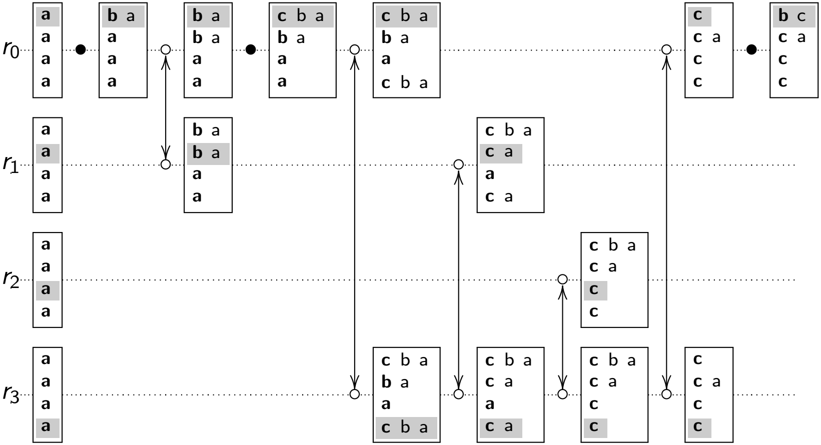
Pointwise order



Since version vectors are define with pointwise order it is enough do find a bounded replacement for each component that defines a total order in all frontiers.

Data Causality

Bounded version vectors



Data Causality

Bounded version vectors

Bounded version vectors can characterize data causality with a state that is independent on the number of updates. The required state is polynomial with respect to the number of replicas.

Let U be the number of updates, and N the number of replicas.

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- Bounded version vectors have scale $O(N^3 \log_2(N))$

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In addition, synchronizations must be bidirectional.

Data Causality
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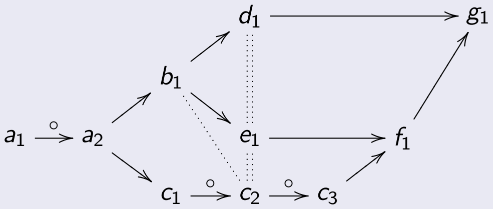

Data Causality
Version Stamps

Identity component

- Local management of the namespace.
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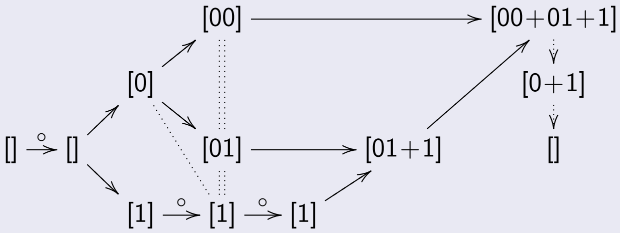
Other features

- Both components are a set of binary strings.
- No map from identifiers to counters is kept.
- No counters are used at all.
- Structure can grow and shrink.

Data Causality

Version Stamps

Identity component



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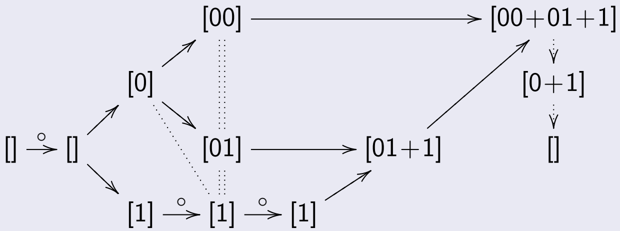
Data Causality

Version Stamps

Identity component



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Data Causality

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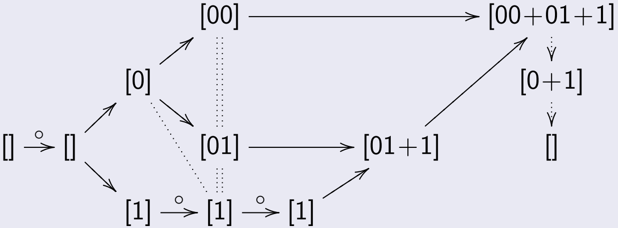

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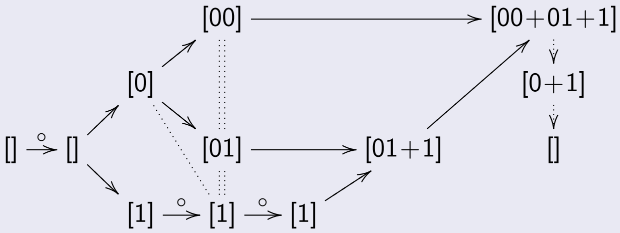

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- Forks append either 0 or 1 to each string in the id.
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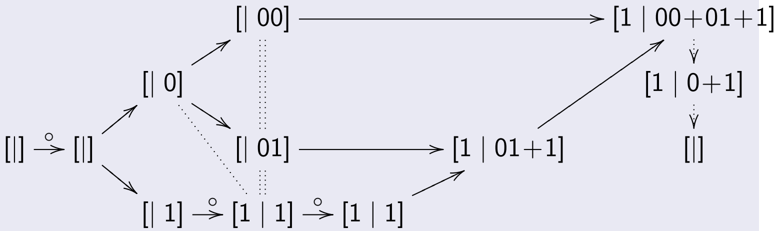
- A possible simplification is attempted upon a join.

Data Causality

Version Stamps

Update component



[|] $\overset{\circ}{\to}$ [||]

[| 0]

[| 1] $\overset{\circ}{\to}$ [1 | 1] $\overset{\circ}{\to}$ [1 | 1]

[| 00]

[| 01]

[1 | 01+1]

[1 | 00+01+1]

[1 | 0+1]

[|]

- Updates copy id into update.

Data Causality

Version Stamps

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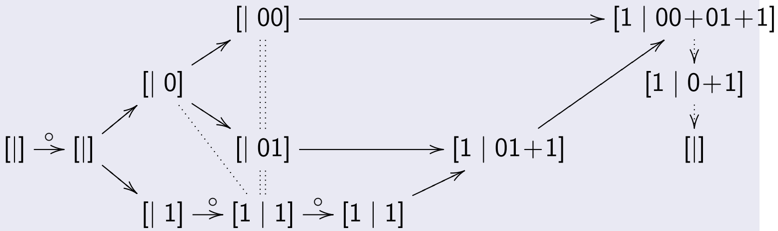

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Data Causality
Version Stamps

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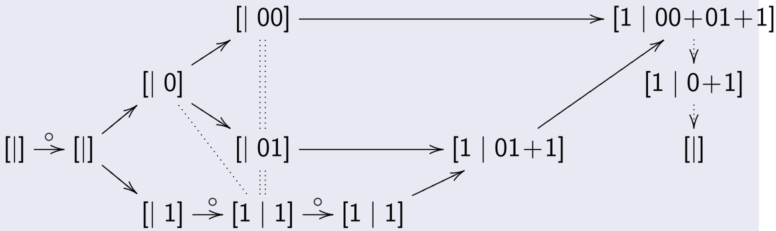

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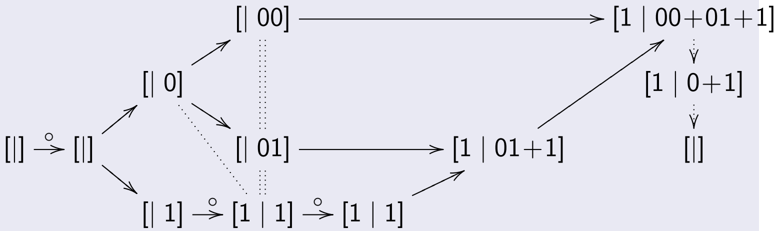
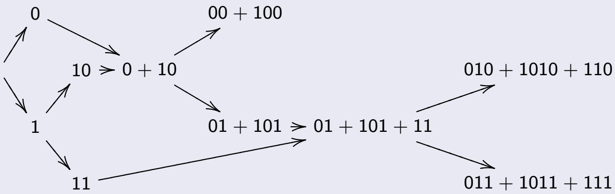


- Updates copy id into update.
- A fork causes no change in the update component.
- A join merges the update components.
- A simplification upon join also reflects in update.

Data Causality

Version Stamps: Pollution of the Namespace

Patologic run



- Pattern: join and fork again with alternating replicas.
- Leads to an overly refined namespace that cannot be simplified.
- This pattern can often occur in a real usage scenario.
- Copy of the identity to the update component aggravates the problem.
- Although correct practical application is severely compromised.

Logical Time and Causality

Function Graphs

Pointwise Order

- $[2, 5, 1] \leq [3, 6, 4]$
- $[2, 5, 1] \not\leq [3, 2, 5]$ and $[3, 2, 5] \not\leq [2, 5, 1]$ means $[2, 5, 1] \parallel [3, 2, 5]$

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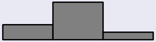
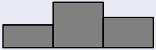
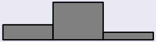
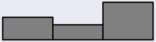
Vector Clocks as Function Graphs

- $[2, 5, 1] \leq [3, 6, 4] \equiv$  $\leq$ 
- $[2, 5, 1] \parallel [3, 2, 5] \equiv$  $\parallel$ 

Function graph containment characterizes causality. Vectors of integers can be re-interpreted as a way to encode the function.

Logical Time and Causality

Stamps

- In order to register new events (advance logical time) each active entity must know which position to update (vector index for that entity).

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Process P_b can hold a stamp $(2, [1, 2, 3])$ giving it access to the 2^{nd} index, and register an update deriving $(2, [1, 3, 3])$.

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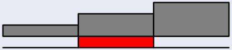
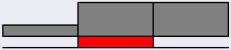
Stamp  can be updated to .

Logical Time and Causality

Global Invariants on IDs

Causality characterization condition

Each entity has a portion of its identity that is exclusive to it. This means each entity having an identity which maps to 1 some element which is mapped to 0 in all other entities.

$$\forall i.\ (i \cdot \bigsqcup_{i' \neq i} i') \neq i.$$

Entity events must use at least a part of the exclusive portion.

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Disjoint condition

A less general but more practical condition is that all identities are kept disjoint. i.e. non-overlapping graphs for any pair of id functions.

$$\forall i_1 \neq i_2.\ \ i_1 \cdot i_2 = \mathbf{0}.$$

Any portion of the id can be used to register events.

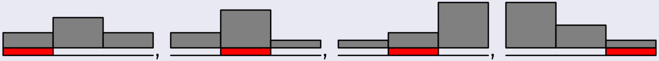
Logical Time and Causality

Plausible Clocks and Lamport Clocks

Plausible clocks [Torres-Rojas 99] and Lamport clocks [Lamport 78] do not meet the causality characterization condition. They are only consistent with causality.

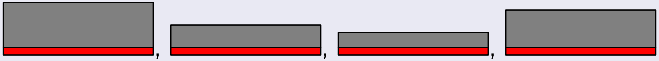
Plausible Clocks

Entities can share ids and update on the same position.



Lamport Clocks

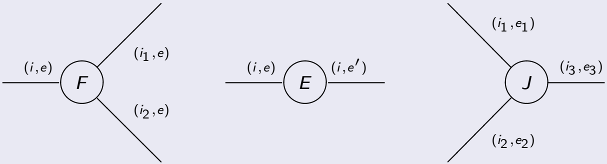
A single id position is used across all entities.



Fork-Event-Join Model

Causality tracking mechanisms (both static and dynamic) can be modeled by a set of core operations: *fork*; *event* and *join*, that act on stamps.

FEJ kernel



Function space based Clock Mechanisms

under the Disjoint Condition

- Core operations:

$$\text{fork} \quad \mathit{fork}((i,e)) \doteq ((i_1,e),(i_2,e))$$
$$\text{subject to } i_1 + i_2 = i \text{ and } i_1 \cdot i_2 = \mathbf{0}.$$

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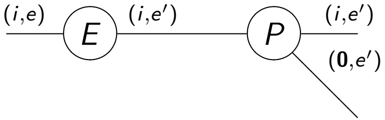
- Peek, a special kind of fork is usefull to create imutable entities (messages or replicas):

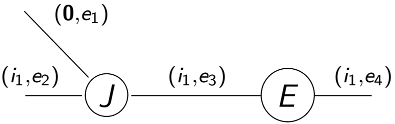
$$\begin{array}{rl} \text{peek} & \mathit{peek}((i,e)) \doteq ((\mathbf{0},e),(i,e)). \end{array}$$

FEJ
Send

Send: This operation is the atomic composition of *event* followed by *peek*. E.g. in vector clock systems, message sending is modeled by incrementing the local counter and then creating a new message.

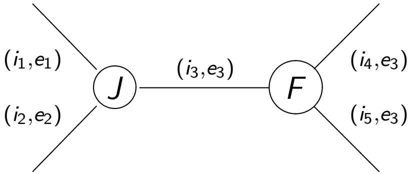


Receive: A *receive* is the atomic composition of *join* followed by *event*. E.g. in vector clocks taking the pointwise maximum is followed by an increment of the local counter.



Apendix
Sync

Sync: A *sync* is the atomic composition of *join* followed by *fork*. E.g. In version vector systems and in bounded version vectors it models the atomic synchronization of two replicas.



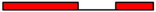
- ITC is a concrete mechanism that meets the FEJ specification.
- Allows decentralized creation and retirement of entities.
- The representation adapts automatically to the number of existing entities, growing or shrinking appropriately.
- ITC is based on functions over a continuous infinite domain ($\mathbb{R}$) with emphasis on the interval $[0, 1)$
- Functions are encoded as trees.

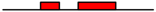
Interval Tree Clocks

Id Component

The id component is an *id tree* with the recursive form:

$$i \doteq 0 \mid 1 \mid (i_1, i_2).$$

$(1,(0,1)) \quad \sim$ 

$((0,(1,0)),(1,0)) \quad \sim$ 

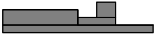
Interval Tree Clocks

Event Component

The event component is a binary *event tree* with non-negative integers in nodes:

$$e \doteq n \mid (n, e_1, e_2).$$

$$(1, 2, (0, (1, 0, 2), 0)) \quad \sim$$

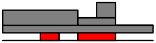


Interval Tree Clocks

ITC Stamps

A stamp in ITC is a pair (i, e).

$$(((0,(1,0)),(1,0)),(1,2,(0,(1,0,2),0))) \quad \sim$$



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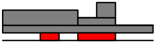
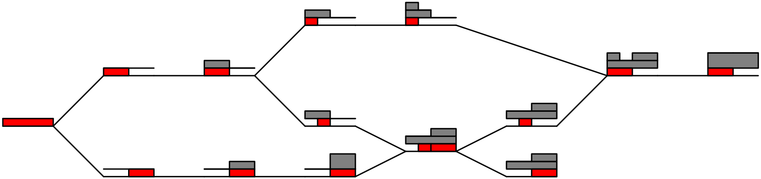


ITC makes use what we call the *seed* stamp, $(1, 0)$, from which we can fork as desired to obtain an initial configuration.

$$(1,0) \quad \sim$$

Interval Tree Clocks

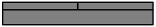
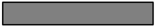
A run

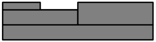
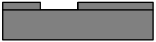


Interval Tree Clocks

Normal form

The event component can be normalized, preserving its interpretation as a function.

$(2, 1, 1) \sim$  $\equiv$  $\sim 3,$

$(2, (2, 1, 0), 3) \sim$  $\equiv$  $\sim (4, (0, 1, 0), 1).$

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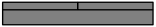
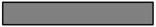
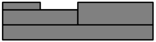
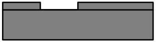
Normal form

The event component can be normalized, preserving its interpretation as a function.

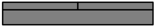
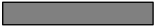
$(2,1,1) \sim$  $\equiv$  $\sim 3,$

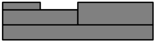
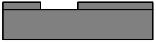
$(2,(2,1,0),3) \sim$  $\equiv$  $\sim (4,(0,1,0),1).$

- Normalization helps to keep a compact encoding.

Interval Tree Clocks
Normal form

The event component can be normalized, preserving its interpretation as a function.

$(2, 1, 1) \sim$  $\equiv$  $\sim 3,$

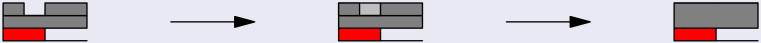
$(2, (2, 1, 0), 3) \sim$  $\equiv$  $\sim (4, (0, 1, 0), 1).$

- Normalization helps to keep a compact encoding.
- Counters flow from leaves to root, further helping encoding.

Interval Tree Clocks

Event operation

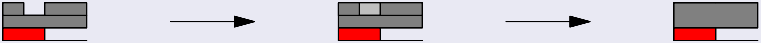
Fill



Interval Tree Clocks

Event operation

Fill

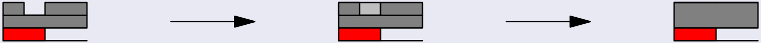


Fills can fill several areas and induce large simplifications.

Interval Tree Clocks

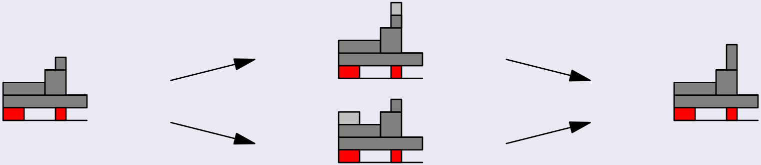
Event operation

Fill



Fills can fill several areas and induce large simplifications.

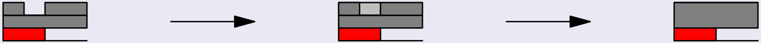
Grow



Interval Tree Clocks

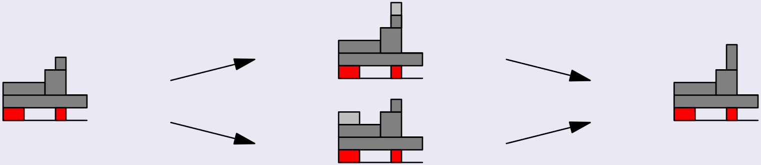
Event operation

Fill



Fills can fill several areas and induce large simplifications.

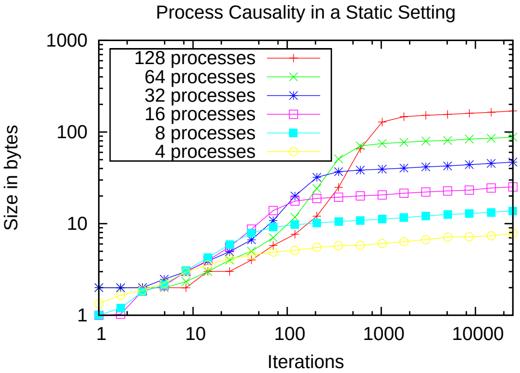
Grow



Fills are prefered to Grows, and alternative Grows are selected by evaluating its impact on encoding size.

Interval Tree Clocks

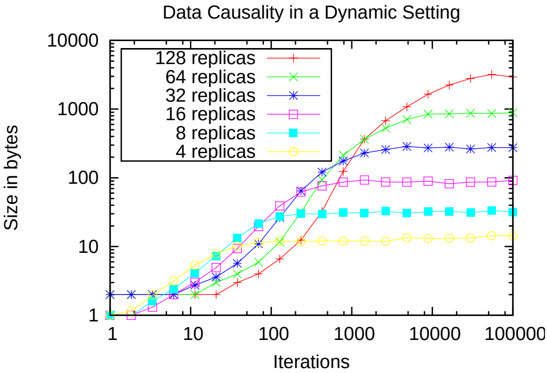
Fixed set of processes exchanging messages



For process causality in a static scenario, we operate on a fixed set of processes doing message exchanges (via peek and join) and recording internal events; here ids remain unchanged, since messages are anonymous.

Interval Tree Clocks

Data replicas under churn



Each iteration consists of forking, recording an event and joining two replicas, each performed on random replicas, leading to constantly evolving ids. This pattern maintains the number of existing replicas while exercising id management under churn.

Bibliography

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- Interval Tree Clocks: A Logical Clock for Dynamic Systems. OPODIS 2008